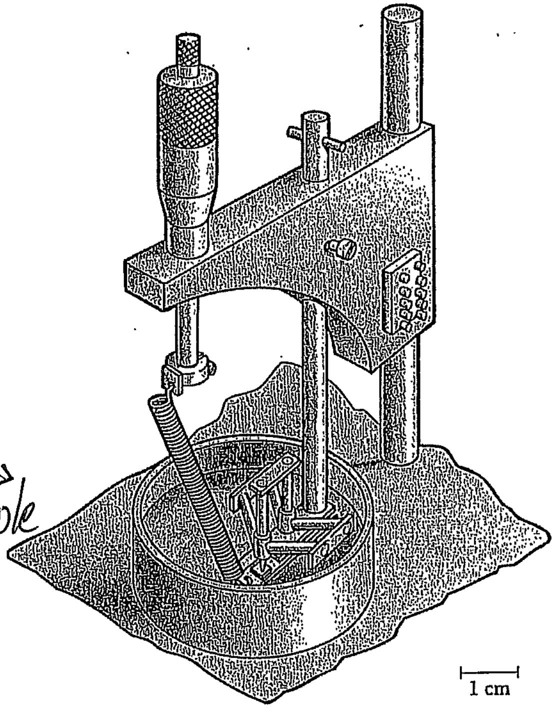
E. More applications of Tunneling

(a) Scanning Tunneling Microscope

[Binnig, Rohrer, Gerber, Weibel, Phys. Rev. Lett. 49, 57 (1982)]

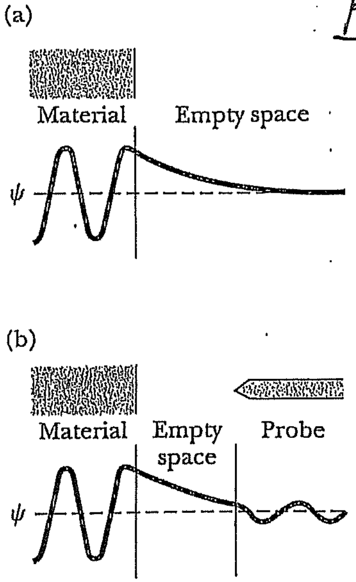
$\underbrace{\text{Binnig, Rohrer}}_{\text{1986 Nobel Physics Prize}}$

a device to maintain distance between tip and sample



One design for a scanning tunneling microscope (STM). The sample to be studied is mounted on a plate in the cylindrical dish. The probe extends beneath the left tripod. The micrometer attached to the spring is used to position the sample.

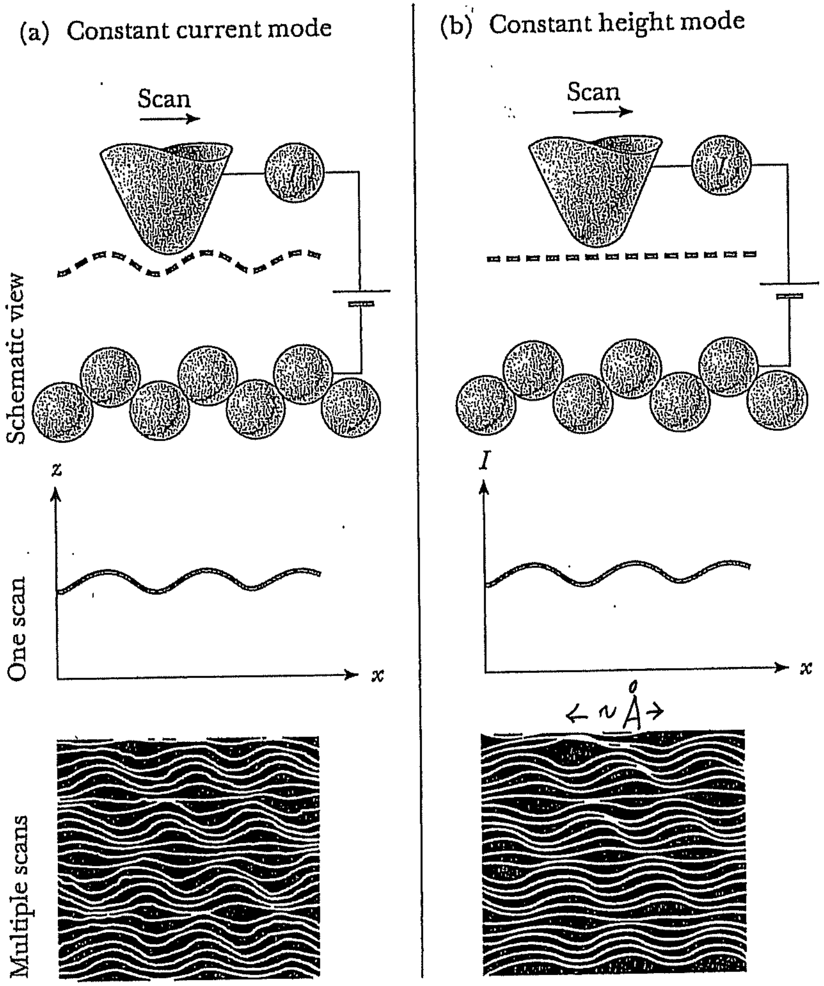
[From Serway et al., "Modern Physics"]



(a) The wavefunction of an electron in the surface of the material to be studied. The wavefunction extends beyond the surface into the empty region. (b) The sharp tip of a conducting probe is brought close to the surface. The wavefunction of a surface electron penetrates into the tip, so that the electron can "tunnel" from surface to tip.

Recall: Work function is a barrier for electrons in metal to get out

Put a sharp tip very close to the surface of a (conducting) material



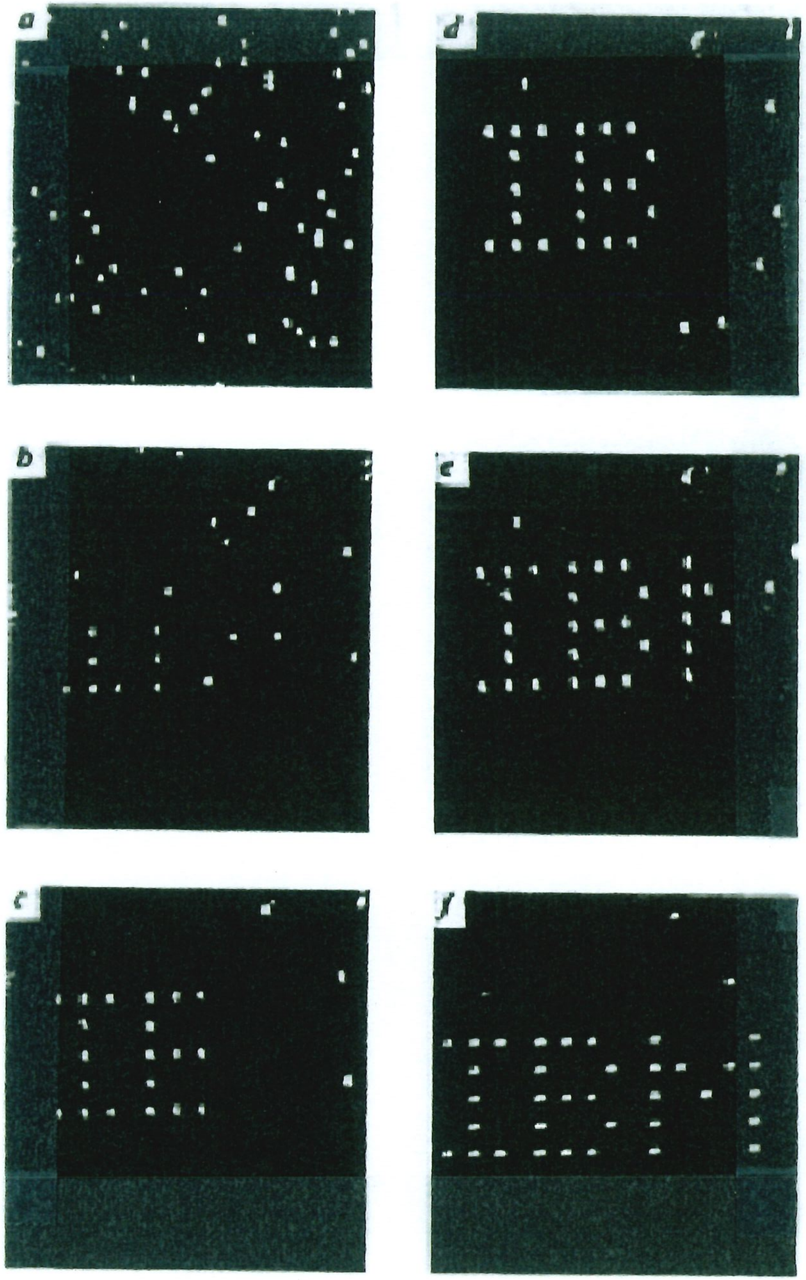
Recall:
T is exponentially sensitive to L, control on height must be done to high accuracy

← "Picture" of atoms

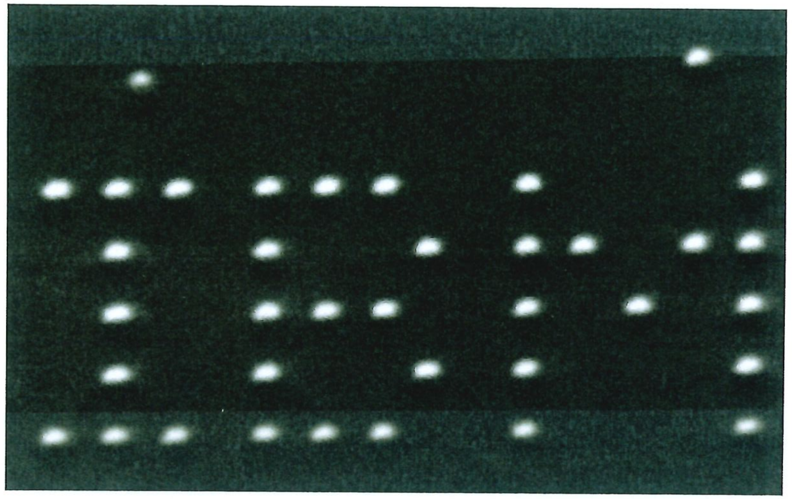
Scanning tunneling microscopes can be operated in either (a) the constant current mode or (b) the constant height mode. The images of the surface of graphite were made by Richard Sonnenfeld at the University of California at Santa Barbara. The constant height mode was first used by A. Bryant, D. P. E. Smith, and C. F. Quate, *Appl. Phys. Lett.* 48:832, 1986.

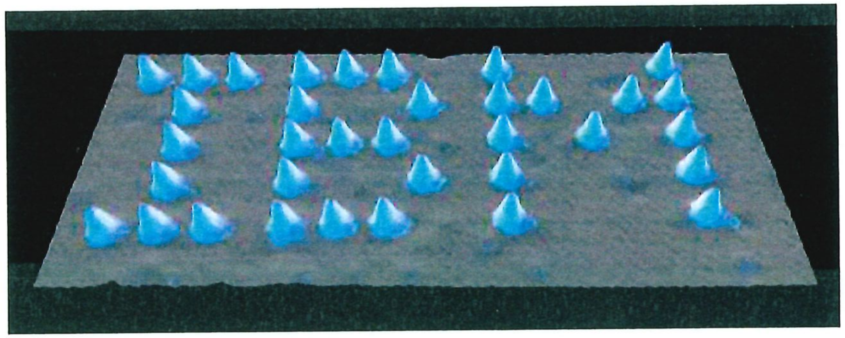
See Nobel Lecture in Review of Modern Physics 59, 615 (1987)

Tip of STM can be used to move adatoms around on a surface



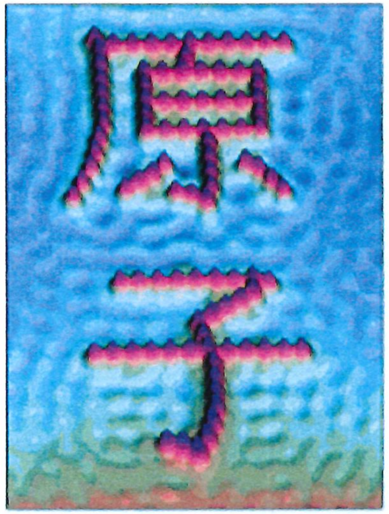
Eigler (IBM) (1989) wrote IBM using 35 Xenon atoms





And they can write Chinese too

Writing With Atoms



Credit: IBM Research

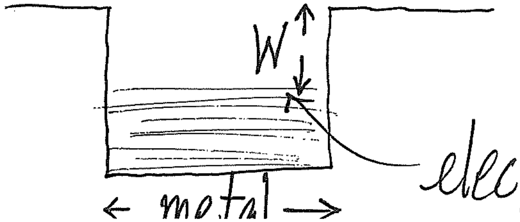
STM : Tool helped develop materials Science [nanoscience],
see and manipulate atoms

(b) Field Emission (from metals)

- J. J. Thomson's Cathode ray tube
- Old TV sets
- Early-day electronic devices
- Many important experiments in the development of physics that used electric discharge

- Work function: Minimal energy an electron needs to get out of a metal

  ∴ Work function is a barrier (at the surface) for electrons

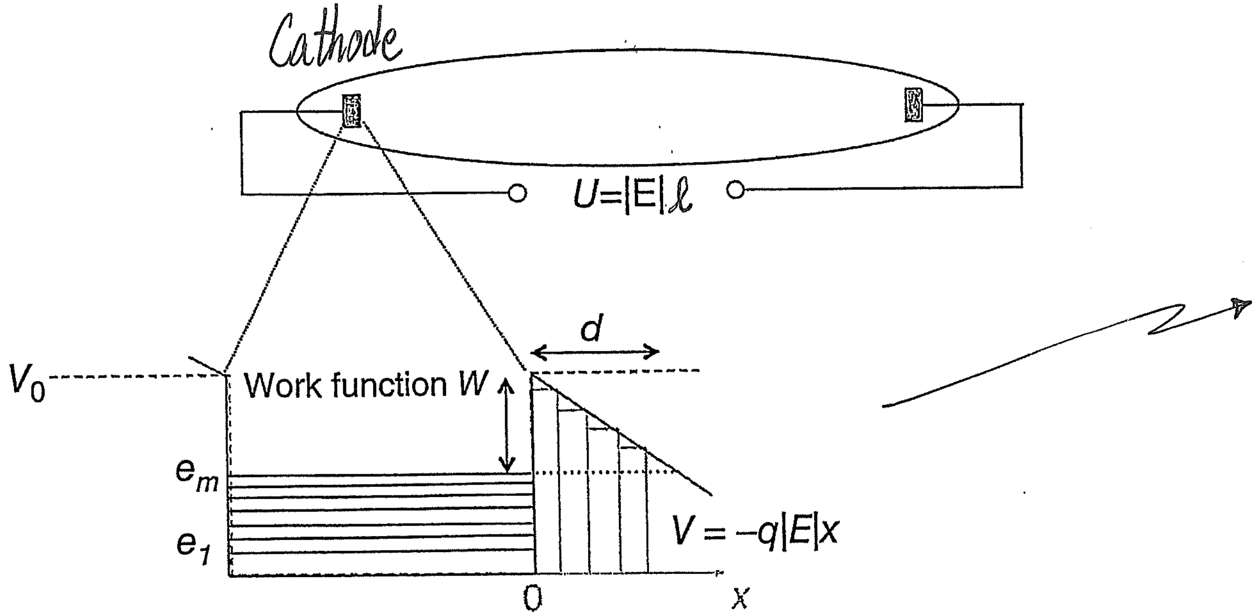


W = Work function

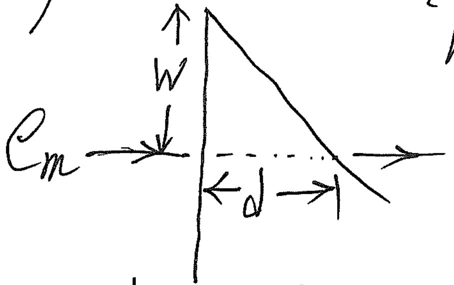
electrons occupy states up to here

- To help electrons get out (and watch TV), apply a field (electric field) [field → electric potential → electric potential energy]

Field Emission



For electrons at e_m (highest energy), they see



Tunneling!

triangular barrier with d depending on $|E|$

[applied field ⇒ electric p.e. linear in x]

- bigger $|E|$, shorter d to go through

$$q|E|\underbrace{d}_{\uparrow \atop \text{across "d"}} = \underbrace{W}_{\uparrow \atop \text{dropped } W} \quad \Rightarrow \quad \underset{\uparrow \atop \text{barrier width}}{d} = \frac{W \leftarrow \text{Work function}}{q|E|}$$

Recall: $T \sim e^{-2\int_0^d \sqrt{\left(\frac{2m}{\hbar^2}\right)[V(x)-e_m]}\,dx}$

Roughly, expects to see $\sqrt{W}$ and $\overset{W/q|E|}{d}$ in the exponential at the end

$$\overset{\text{"charge"}}{\text{Current}} \text{ Density } J = J_0 \exp\left(-\frac{\sqrt{32m}}{3\hbar}\,\frac{W^{3/2}}{q|E|}\right)$$

field emission
(1928)

[$\underbrace{\text{Fowler}}_{\text{[PhD advisor of Dirac]}}$-Nordheim tunneling formula]

- Key to understand STM [tip with applied potential difference]
- Note: effect of temperature is not considered here

heating up the electrode

if heating effect is important, then $J \sim e^{-W/kT}$

thermal physics

(1901)

for thermionic emission

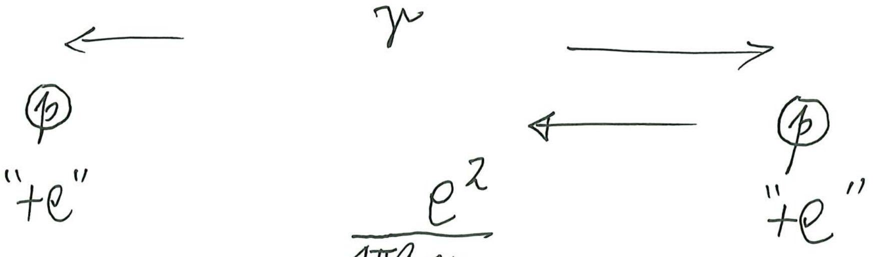
[called Richardson's law]

[1928 Nobel Prize]

(c) Our existence : Fusion energy from the Sun (Stars)

How hydrogen & hydrogen come together to form helium?

OR how proton & proton come together to form helium nucleus?

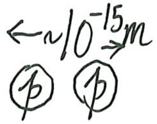


$$\underbrace{\frac{e^2}{4\pi\varepsilon_0 r}}$$ ← increases as p - p approach

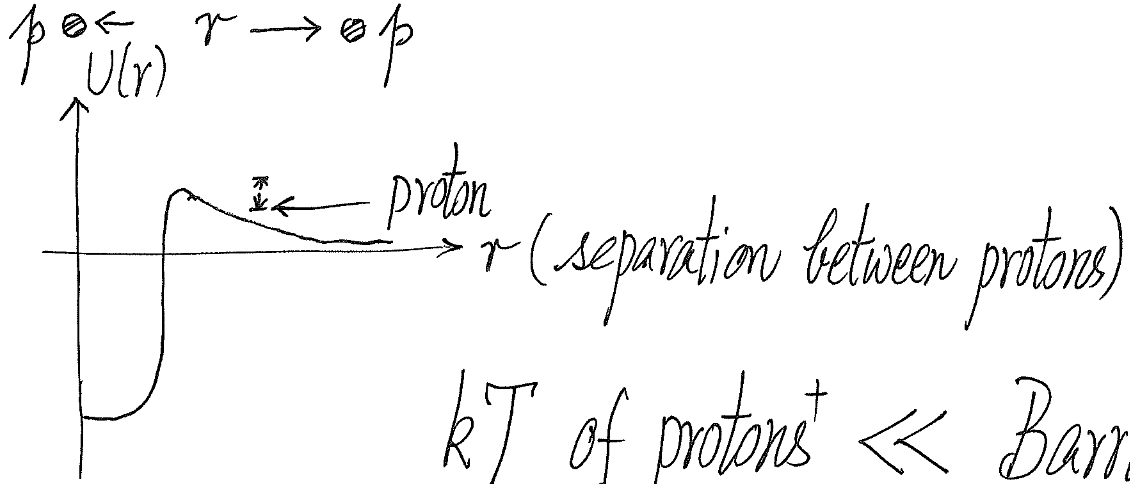
$$\sim \frac{1.44}{r}\ (\text{MeV}\cdot\text{fm}) \qquad [\text{fm} \sim 10^{-15}\,\text{m}] \qquad r = 1\,\text{fm},\ \underbrace{1.44\,\text{MeV}}_{}$$

$T \sim 10^{10}\,\text{K}$

⇒ Coulomb potential energy barrier to prevent p approaches p



Only when p-p are sufficiently close, then Nuclear force becomes effective (short-range), and p-p want to bind



kT of protons[†] $\ll$ Barrier to overcome

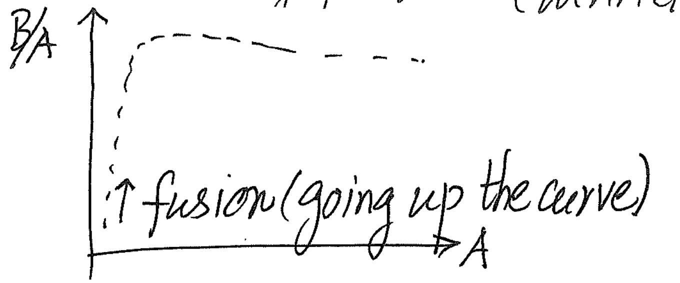
- Similar situation for the other steps
- How could fusion happen?

Tunneling! [One reason why prob. per collision is so tiny]

(another reason is weak interaction)

[†] Temperature at center of Sun $\sim 1.58 \underbrace{\times 10^7\,\text{K}}_{\text{\underline{too low} to surpass barrier}}$

The energy from the Sun comes from fusion of protons (tunneling)



<u>pp cycle (proton-proton cycle)</u>

(i) $${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + \underbrace{e^{+} + \nu_e}_{\text{"}\beta^{+}\text{ decay"}} \quad (+0.4\ \text{MeV})$$
$(p + p)$ [weak interaction ⇒ extremely unlikely process]

(ii) $${}^{2}_{1}H + {}^{1}_{1}H \rightarrow {}^{3}_{2}He + \gamma + (5.5\ \text{MeV})$$

(iii) $${}^{3}_{2}He + {}^{3}_{2}He \rightarrow {}^{4}_{2}He + {}^{1}_{1}H + {}^{1}_{1}H + (12.9\ \text{MeV})$$

"solar neutrinos"

Net effect is: $$4\,{}^{1}_{1}H \rightarrow {}^{4}_{2}He + \underbrace{2e^{+}}_{\text{annihilate with } 2e^{-} \text{ and give } \underline{2\text{MeV}}} + \overline{2\nu_e}\ (+\ \underline{24.7}\ \text{MeV})$$

$\therefore$ 26.7 MeV for $4\,{}^{1}_{1}H$ fuse into ${}^{4}_{2}He$

But tunneling occurs with tiny probability.

That's good! The Sun burns slowly, just right to keep us alive!

The Sun is burning very slowly (in terms of per proton)!

- Over 4.6×10^9 years (age of the Sun), hydrogen reduced by ~50%
  - But number density of protons[†] in huge $n \sim 6 \times 10^{25}\ \mathrm{cm}^{-3}$

    $\Rightarrow$ p & p collide frequently ($\sim 10^{12}$ times per second)
- From its luminosity, the success rate of step (i) is <u>tiny</u> (weak interaction)

  prob. of fusion per collision $\sim 2 \times 10^{-31}$ $\leftarrow$

[†] In metals, the number density of electrons $\sim 10^{23}\ \mathrm{cm}^{-3}$

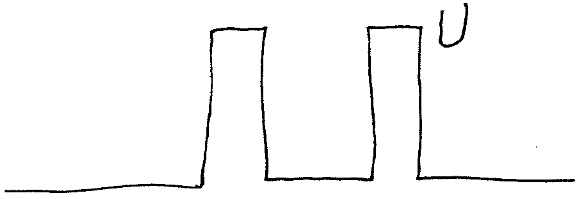
And controlled Fusion as future energy source?

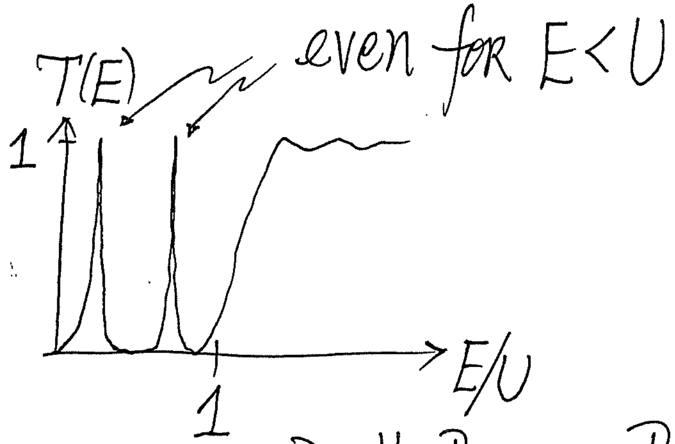
How to overcome the 10^{10} K barrier?

See iter.org

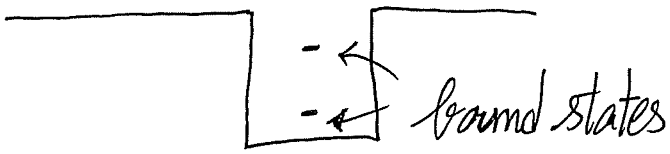
on-going international project to produce NET energy from fusion for the first time

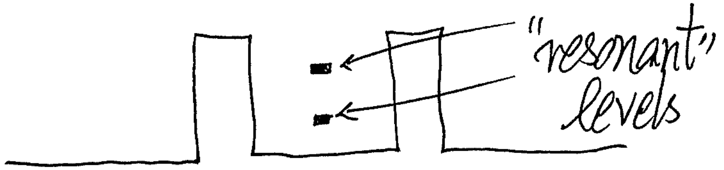
(d) Resonant Tunneling



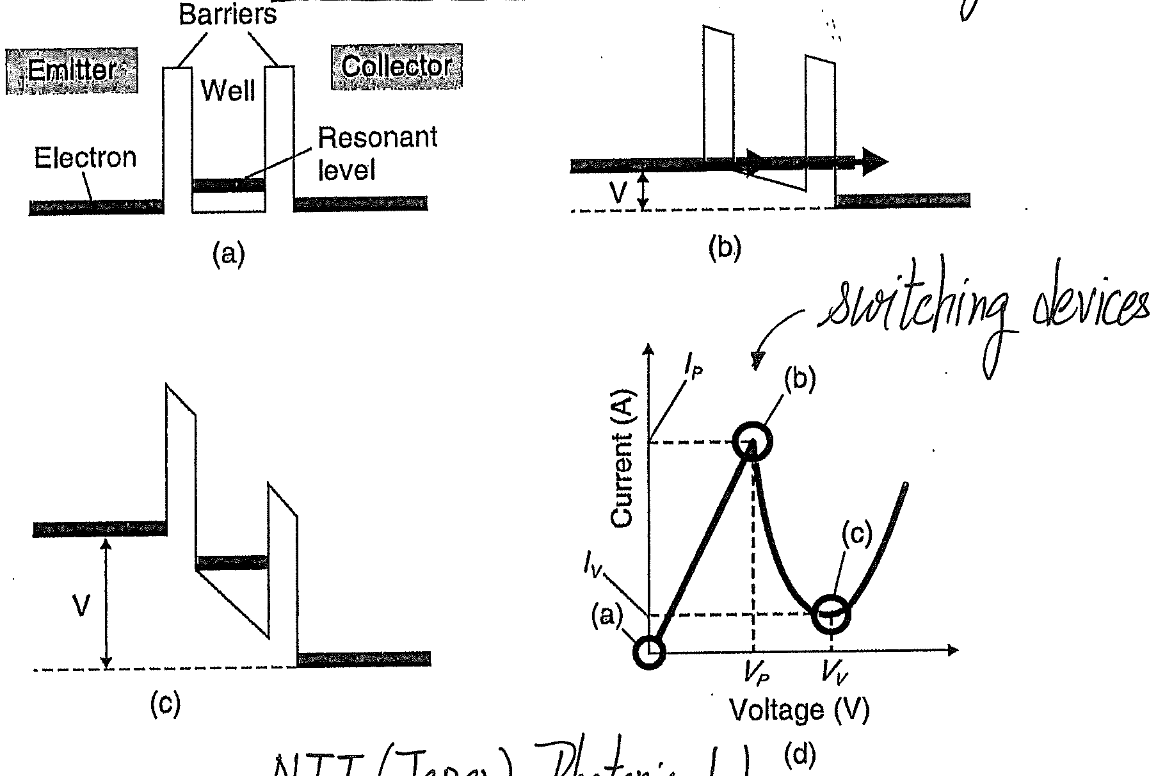


c.f.





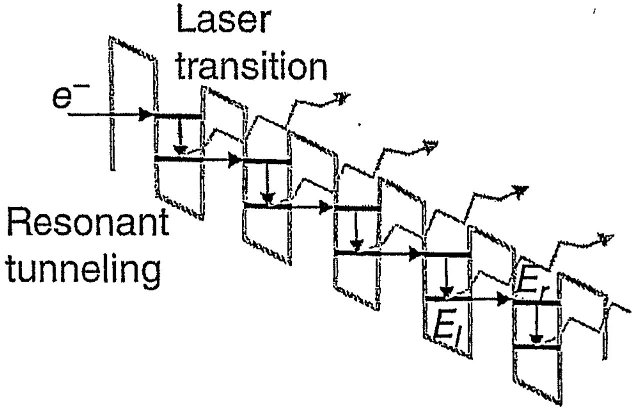
Double Barrier Resonant Tunneling Diode



NTT (Japan) Photonics Lab

Conduction band profiles of a DB RTD at three different bias voltages (V): (a) zero bias ($V=0$), (b) resonance ($V = V_P$), and (c) off-resonance ($V = V_V$). (d) Schematic current-voltage characteristics of a DB RTD.

(e) Quantum Cascade Laser



- Capasso (invented in Bell Lab, now at Harvard)

Other "applications":

- Flash Memory [related to MOS (Metal-$\overset{\text{insulator (barrier)}}{\text{oxide}}$-semiconductor) structure]
- Tunneling limits further shrinking in size of MOS-$\underbrace{\text{FET}}_{\text{field effect transistors}}$

References (Tunneling)

- Griffiths, Sec. 2.7, Secs. 8.1, 8.2 (1st edition)
- Rae, Secs 2.5, 9.1 (3rd edition)

Further Reading

P. Deák, "Essential Quantum Mechanics for Electrical Engineers" (Ch. 10)

References (Nuclear Physics, Fusion)

Zafiratos, Taylor, Dubson, "Modern Physics for Scientists and Engineers" (Chapters)
Williams, "Nuclear and Particle Physics"